Isolating and Estimating Undifferenced GPS Integer Ambiguities

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Introduction

- Undifferenced Ambiguity Resolution has been an elusive goal in GPS processing.
- Recent techniques have been introduced that appear to “show the way”. (All aspects addressed?)
- Concurrently, there have been on-going investigations into the so-called “code-biases”.
- The goal of the presentation is to show how:
  - the “standard model” of undifferenced ionosphere-free observables is <u>sub-optimal</u>; and
  - <u>rigorous modelling</u> of code biases facilitates estimation of integer ambiguities from undifferenced observables.

Standard Observation Equations

$$
\begin{aligned}
C_1 &= \rho + T + I + c(dt^r - dt^s) + b_{C1}^r - b_{C1}^s + \varepsilon_{C1} \\
P_1 &= \rho + T + I + c(dt^r - dt^s) + b_{P1}^r - b_{P1}^s + \varepsilon_{P1} \\
P_2 &= \rho + T + q^2 I + c(dt^r - dt^s) + b_{P2}^r - b_{P2}^s + \varepsilon_{P2} \\
\lambda_1(\Phi_1 + N_1) = L_1 &= \rho + T - I + c(dt^r - dt^s) + b_{L1}^r - b_{L1}^s + \varepsilon_{L1} \\
\lambda_2(\Phi_2 + N_2) = L_2 &= \rho + T - q^2 I + c(dt^r - dt^s) + b_{L2}^r - b_{L2}^s + \varepsilon_{L2}
\end{aligned}
$$

- distinguish between geometric and non-geometric (timing) parameters
- b_*^* represent synchronisation errors between measurements – codes and phases measured <u>separately</u>
- understanding their role is crucial to isolating integer ambiguities from undifferenced carrier phases

Standard Observable Model Re-assessed

$$P_3 = \rho + T + c(dt^r - dt^s) + b^r_{P3} - b^s_{P3} + \varepsilon_{P3}$$

$$L_3 = \rho + T + c(dt^r - dt^s) + b^r_{L3} - b^s_{L3} - \lambda_3 N_3 + \varepsilon_{L3}$$

- singular due to functionally identical clocks & biases
- by combining code clock and bias parameters and retaining common oscillators:

$$P_3 \equiv \rho + T + c(dt^r_{P3} - dt^s_{P3}) + \varepsilon_{P3}$$

$$L_3 \equiv \rho + T + c(dt^r_{P3} - dt^s_{P3}) + A_{P3} + \varepsilon_{L3}$$

$$\text{where } A_{P3} = b^r_{L3} - b^r_{P3} - b^s_{L3} + b^s_{P3} - \lambda_3 N_3$$

- hence, even if ${b_{L3}}^*$ known, ambiguities are not isolated

Pseudorange Clock Datum

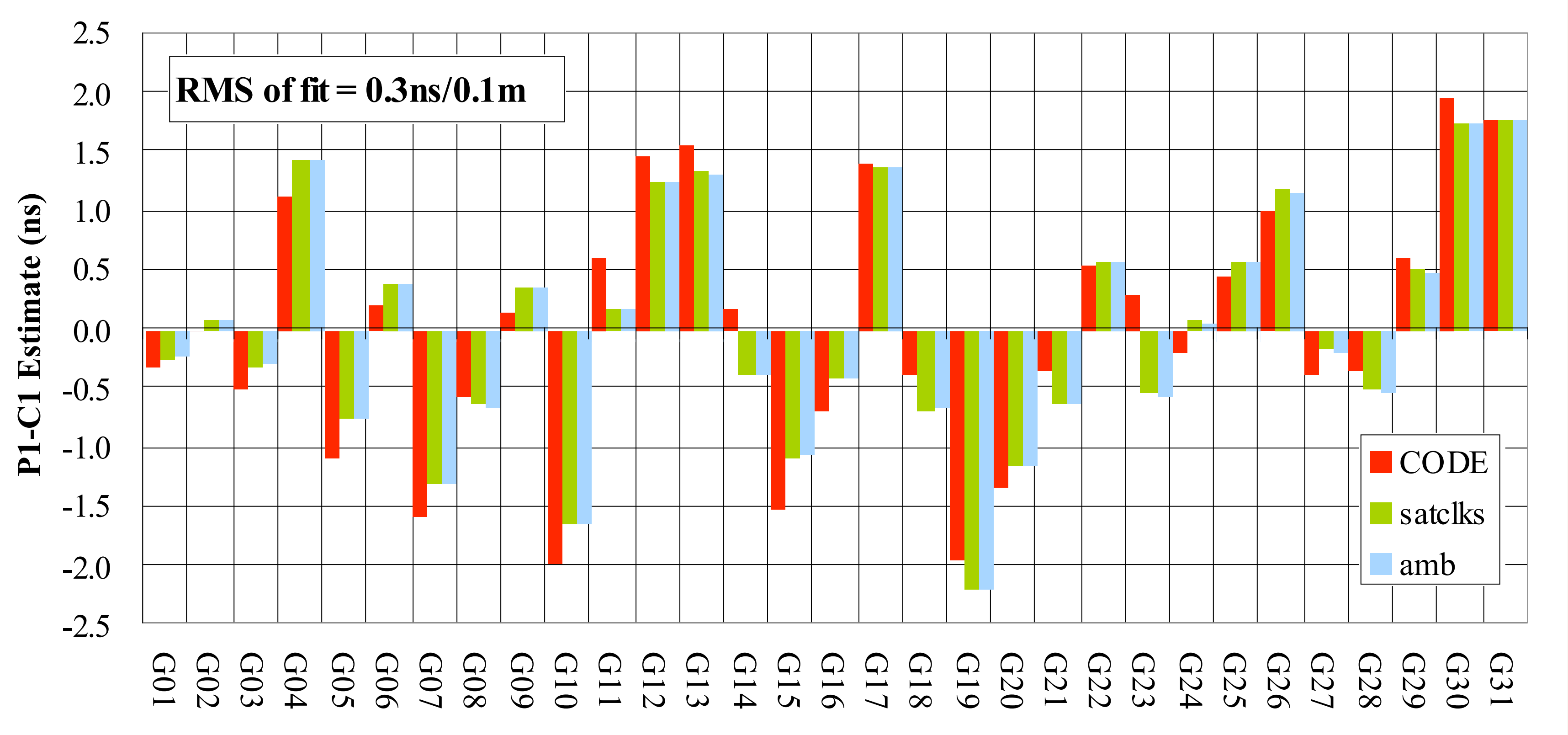
- Because each carrier phase is uniquely ambiguous, the pseudoranges provide the datum for the clock solutions.
- Implication:
  - A change in dual-frequency pseudoranges manifests itself in estimated clocks and ambiguities.
- Example:
  - Compute P1-C1 bias from 2 standard model solutions:

$$P_3 = f(P_1, P_2)$$

$$P_{3'} = f(C_1, P_{2'}) \quad where\ P_{2'} = C_1 + (P_2 - P_1)$$

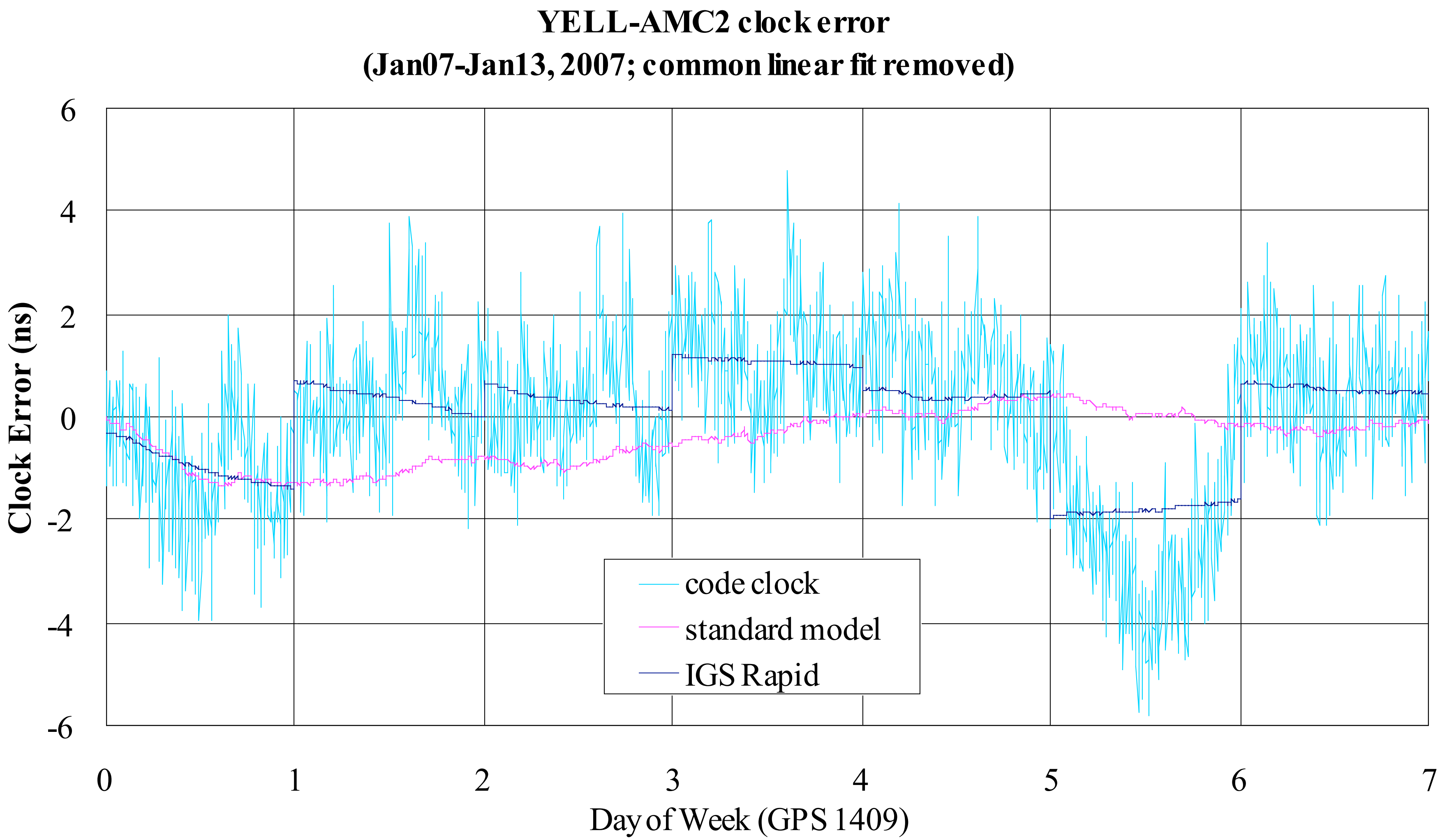
$$dt^s_{P3} - dt^s_{P3'} = \boxed{b^s_{P1-C1}} = A_{P3} - A_{P3'} - b^r_{P1-C1}$$

Deriving satellite P1-C1 biases

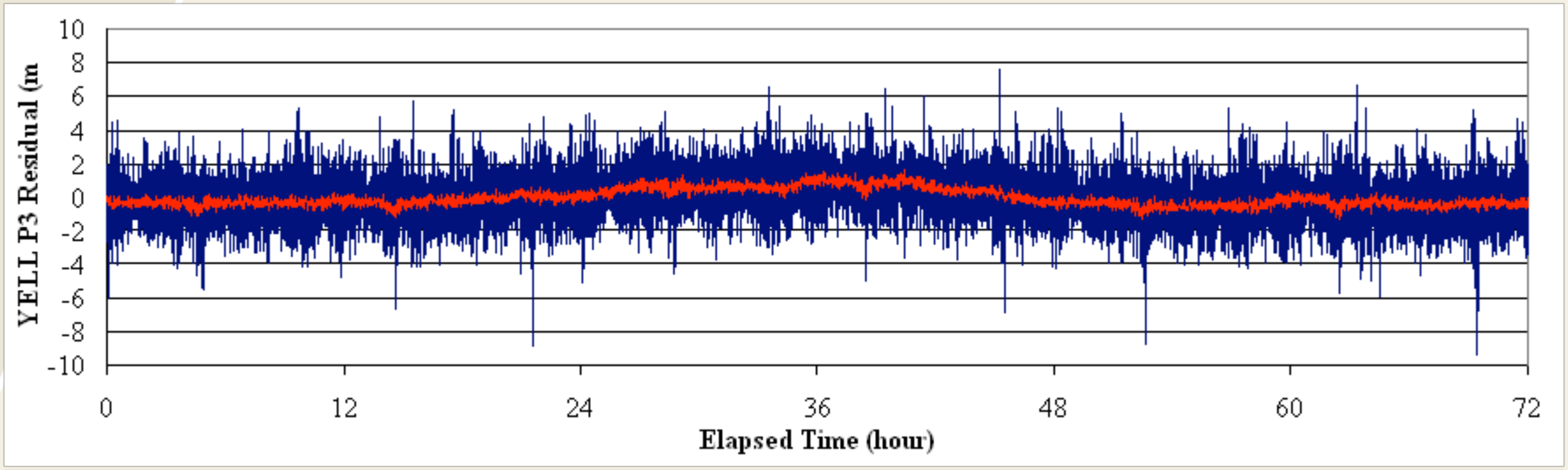


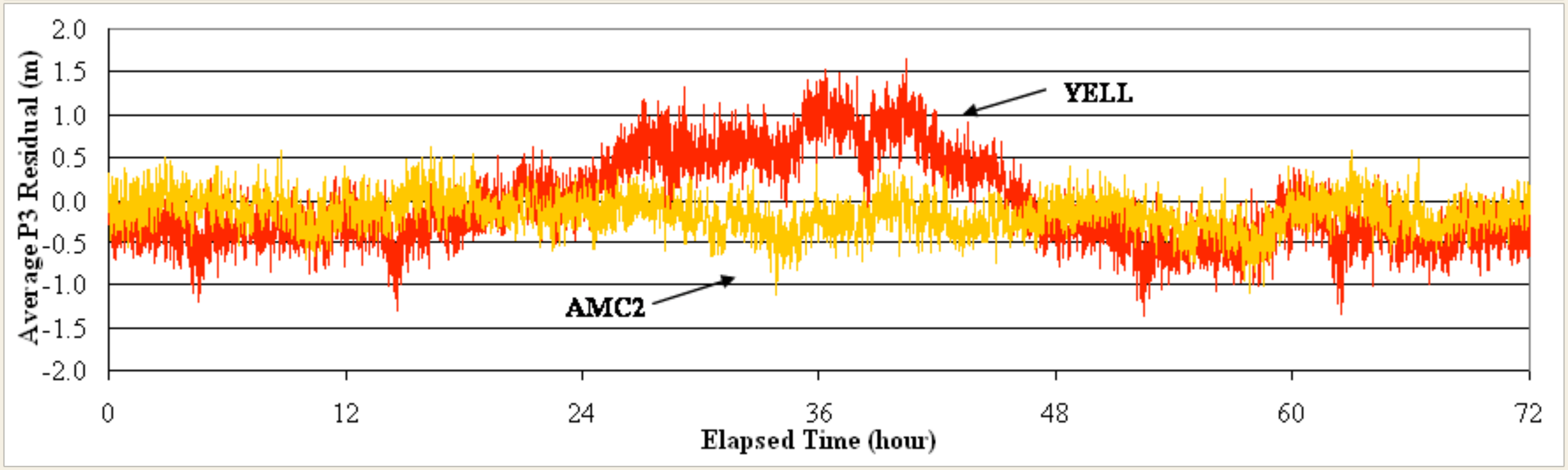
- standard model still optimally parameterised if b_*^* are constant, but…

Day-Boundary Clock Jumps



YELL Residuals – P3 Obs. & Avg.





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Decoupled Clock Model

- Problem:
  - Apparent code and phase oscillator measures are significantly different.
- Solution:
  - Decouple the code and phase clocks:

$$P_3 = \rho + T + c(dt^r_{P3} - dt^s_{P3}) + \varepsilon_{P3}$$

$$L_3 = \rho + T + c(dt^r_{L3} - dt^s_{L3}) - \lambda_3 N_3 + \varepsilon_{L3}$$

  - No assumptions about bias 'stability' required.

Implication:

- Pseudorange datum removed from carrier phase.
- Replace with Ambiguity Datums.

Ambiguity Datum Fixing

- One ambiguity per phase clock fixed, less one
- One phase clock fixed as the network datum
  - Identical concept to fixing the ‘reference clock’ in standard model network processing
  - One code clock fixed also
- Ambiguity can be fixed to arbitrary integer value
  - acts as Partial Integer Constraint
    - remaining ambiguities are integer!
- Phase clock estimates are integer ambiguous with respect to the code clock estimates.

Relationship to Goad Model

- Goad, C.C. (1985). "**Precise Relative Position Determination Using Global Positioning System Carrier Phase Measurements in a Nondifference Mode**." *Proceedings of the First International Symposium on Precise Positioning with the Global Positioning System*. US Dept. of Commerce. Rockville, Maryland, April 15-19. Vol. 1, pp. 347-356.

$$\Phi_1^1(t) = G_1^1(t)/\lambda + B_1^1(t)$$
$$\Phi_1^2(t) = G_1^2(t)/\lambda + B_1^2(t)$$
$$\Phi_2^1(t) = G_2^1(t)/\lambda + B_2^1(t)$$
$$\Phi_2^2(t) = G_2^2(t)/\lambda + N_{12}^{12} + B_2^1(t) + B_1^2(t) - B_1^1(t)$$

→

$$\left.\begin{aligned}\Phi_1^1(t) &= G_1^1(t)/\lambda + B_1^1(t)\\ \Phi_1^2(t) &= G_1^2(t)/\lambda + B_1^2(t)\end{aligned}\right\}\text{network datum}$$
$$\Phi_2^1(t) = G_2^1(t)/\lambda + B_{21}(t) + B_1^1(t)$$
$$\Phi_2^2(t) = G_2^2(t)/\lambda + B_{21}(t) + N_{12}^{12} + B_1^2(t)$$

base-station–base-satellite ➔ datum fixing

4 observations : 4 clk/amb unknowns when $G(t)$ known

$G(t)$: a-priori values or pseudorange estimates

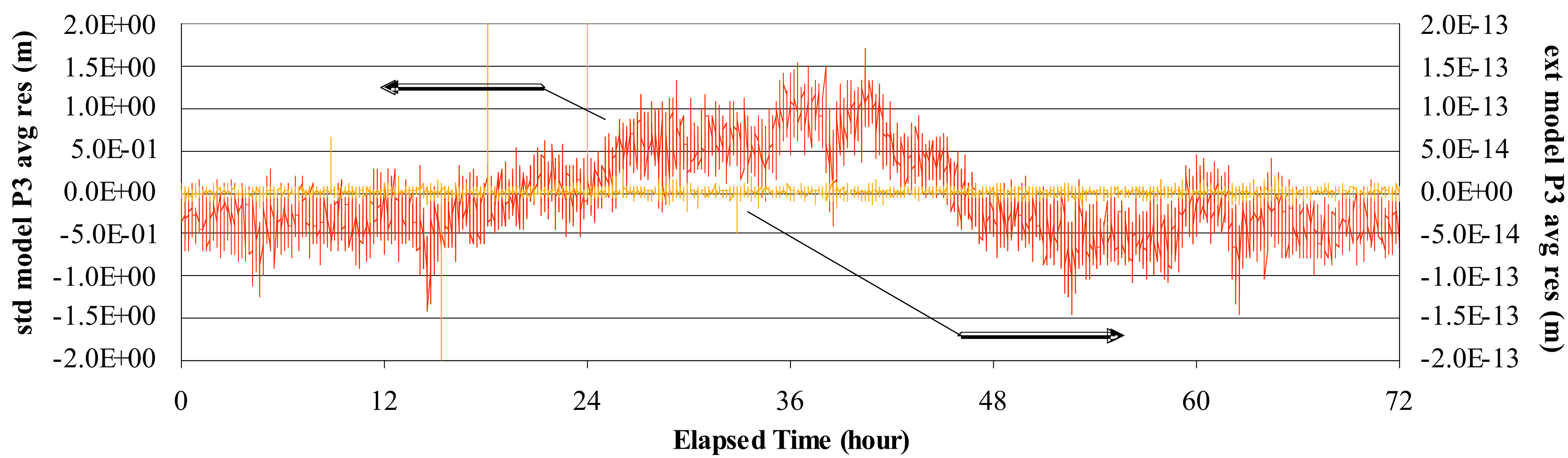
Extended Model

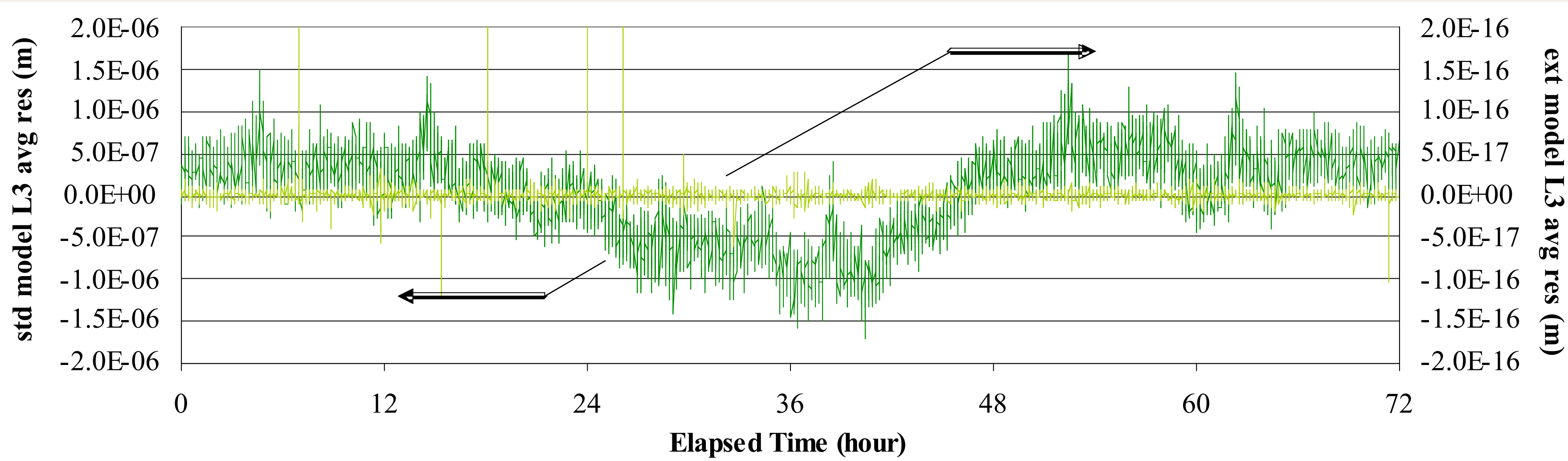
- Problem:
  - λ_{IF}(L1,L2) ≈6mm. (Note: λ_{IF}(L2,L5) ≈12cm!)
- Implication:
  - Intermediate step required for L1,L2 processing
- Solution:
  - Melbourne-Wübbena combination for WL

$$A_4 = L_4 - P_5 = b^r_{A4} - b^s_{A4} - \lambda_4 N_4 + \varepsilon_{A4}$$

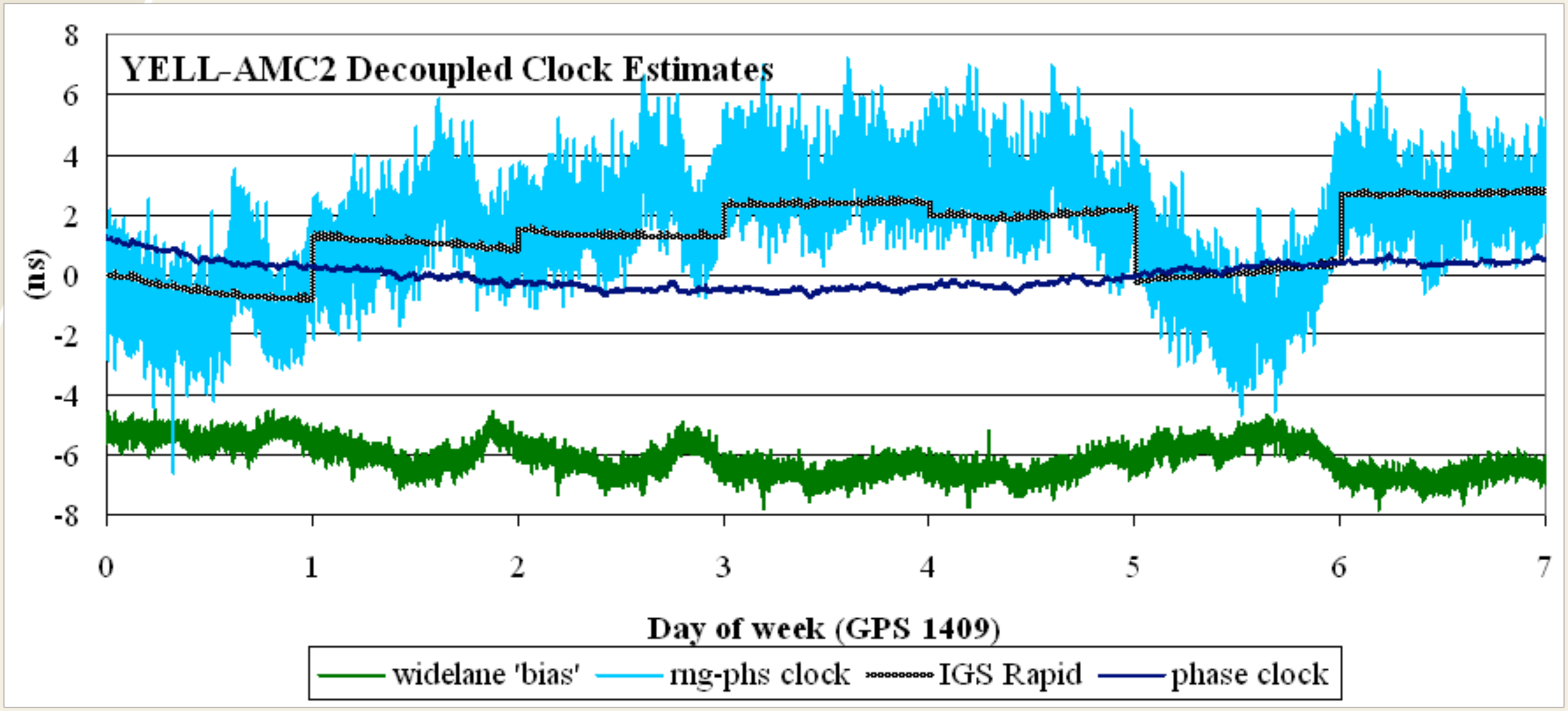
  - b^*_{A4} are <u>not</u> constant – 'delta-clocks'
  - Ambiguity Datum fixing
  - <u>Processed simultaneously</u> with P_3 and L_3
- With WL fixed, λ_{IF}(L1,L2) = λ_{NL}≈11cm

YELL P3 & L3 Average Residuals



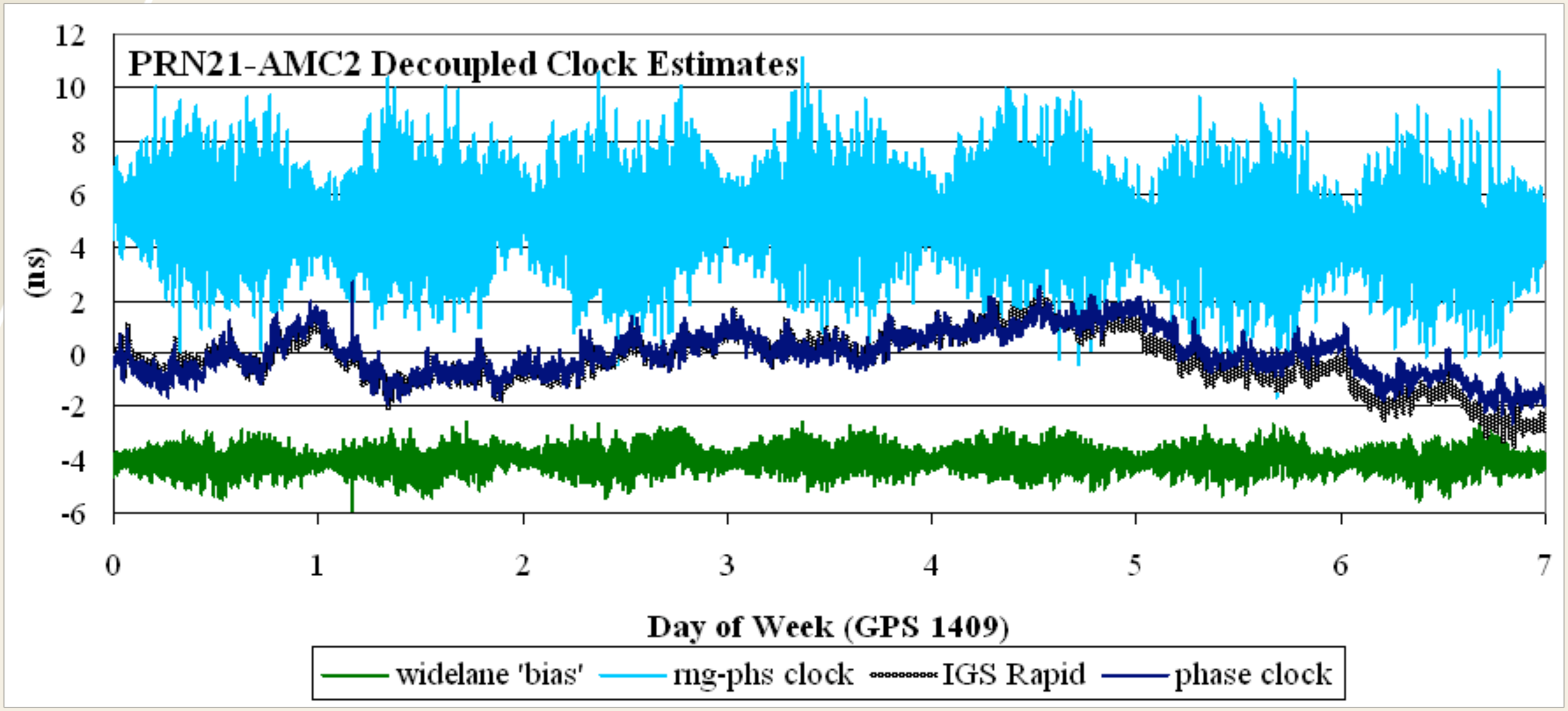


Station Clock Parameter Estimates



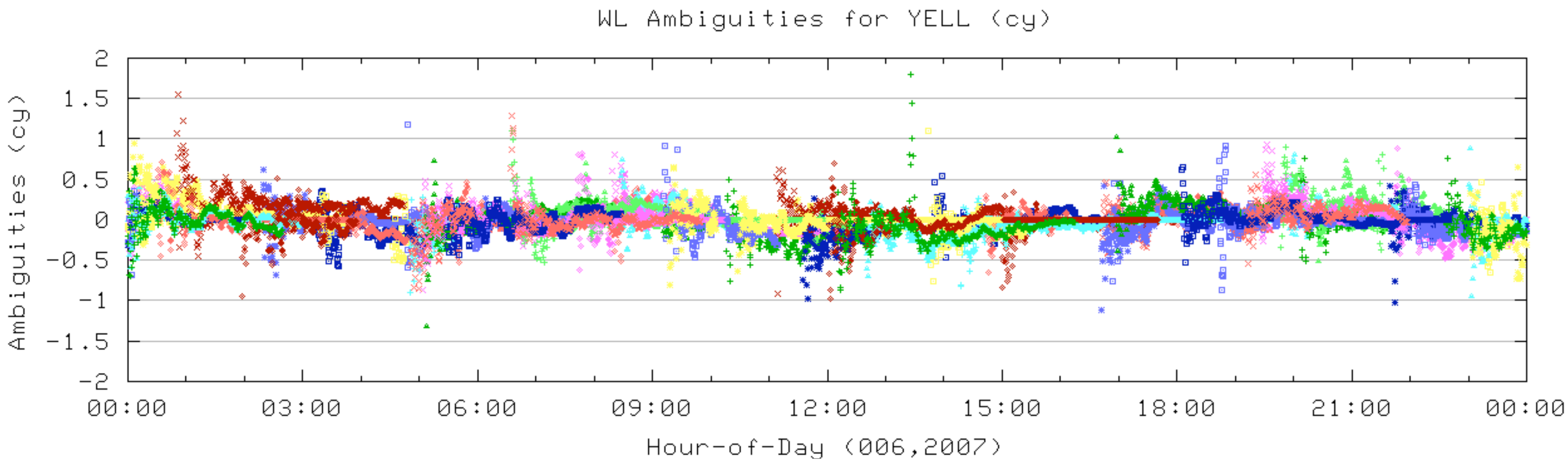
Rapid/phase clock de-trended RMS = 0.08ns/0.02m

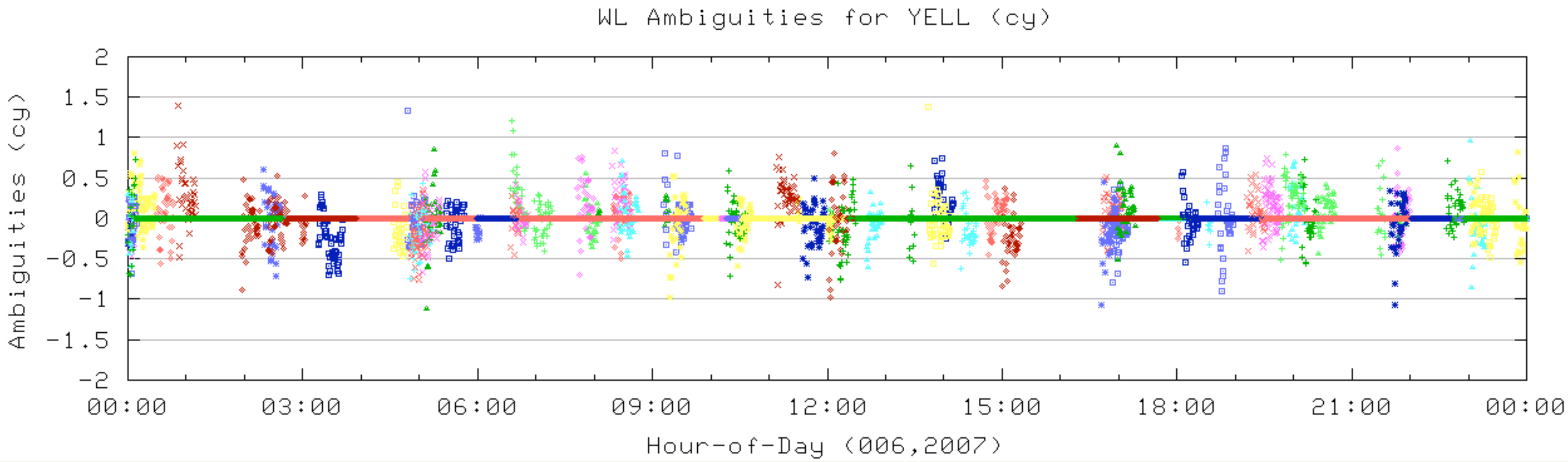
Satellite Clock Parameter Estimates



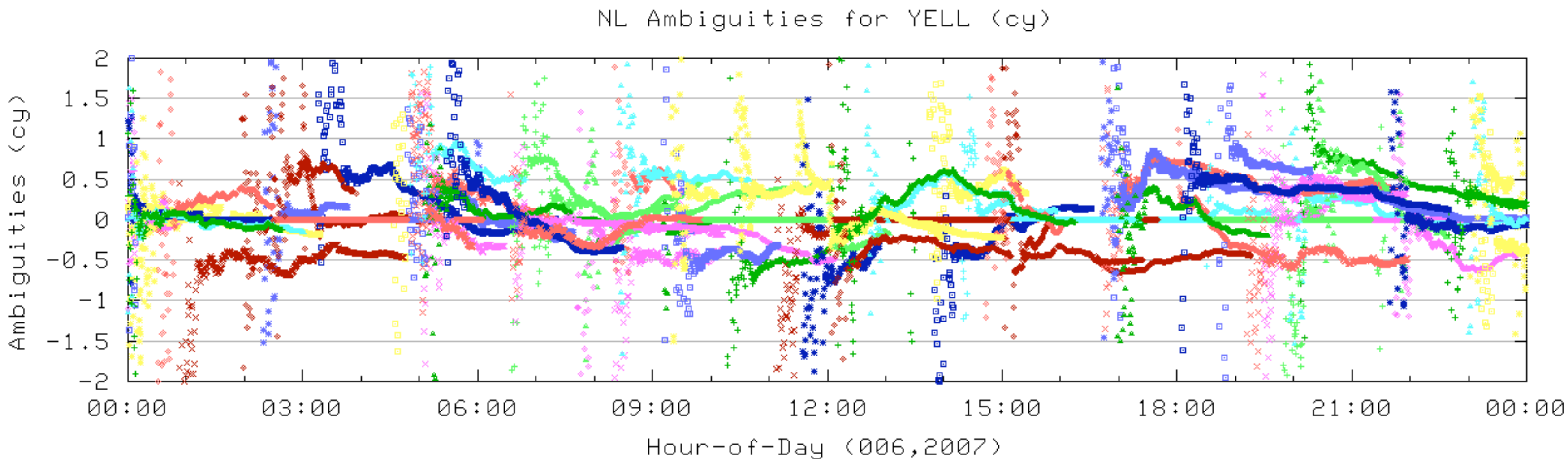
Rapid/phase clock de-trended RMS = 0.17ns/0.05m

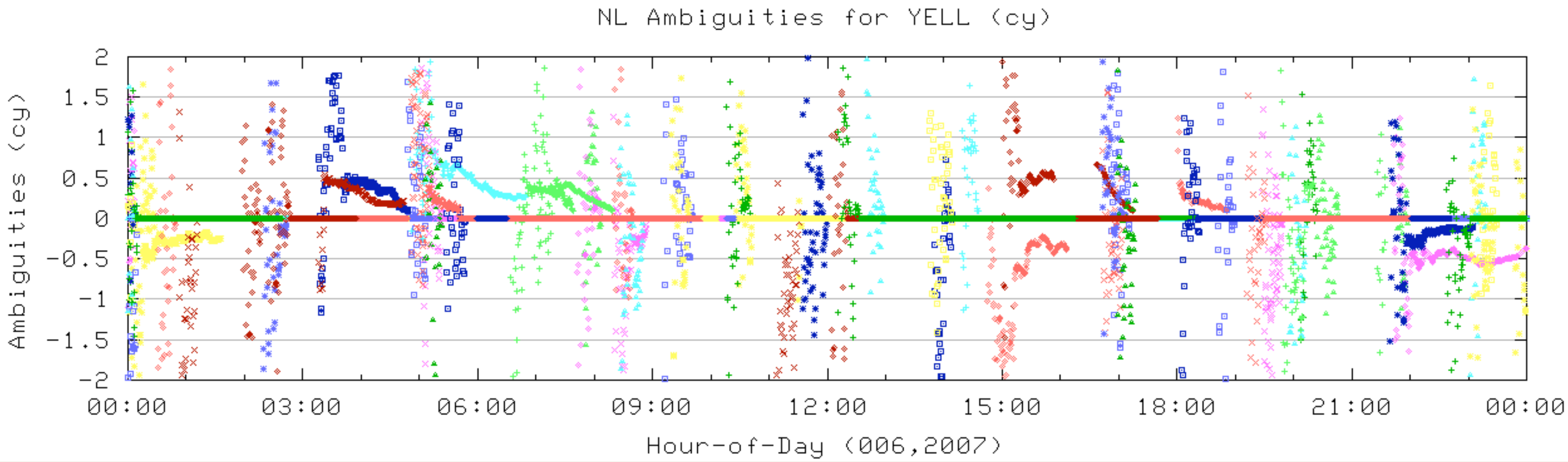
Widelane Ambiguities – Float & Fixed



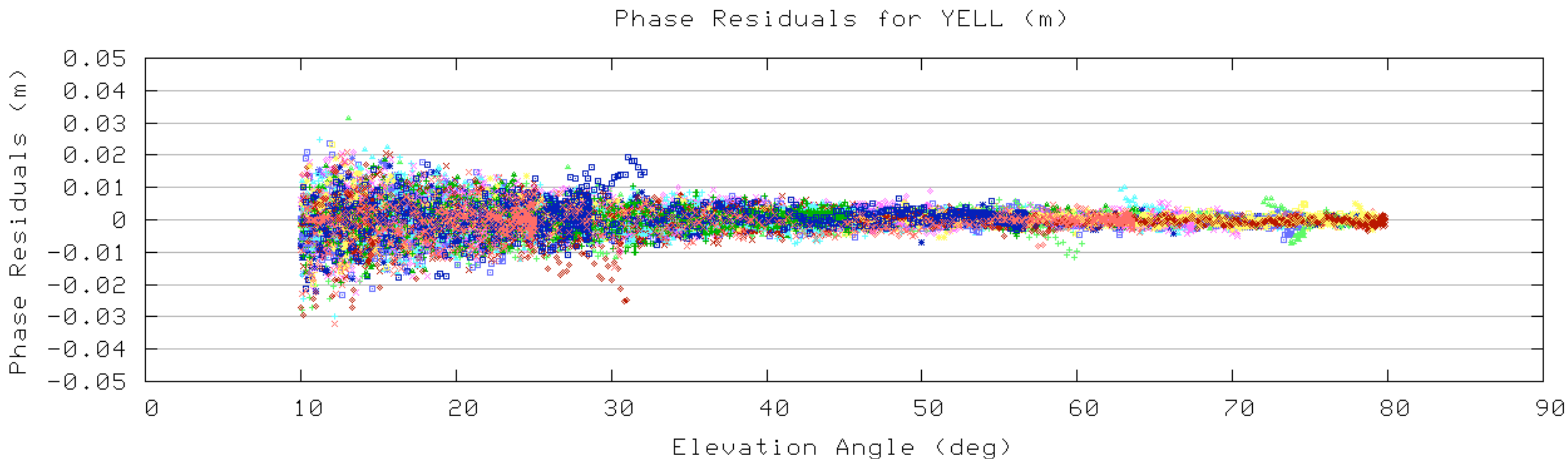


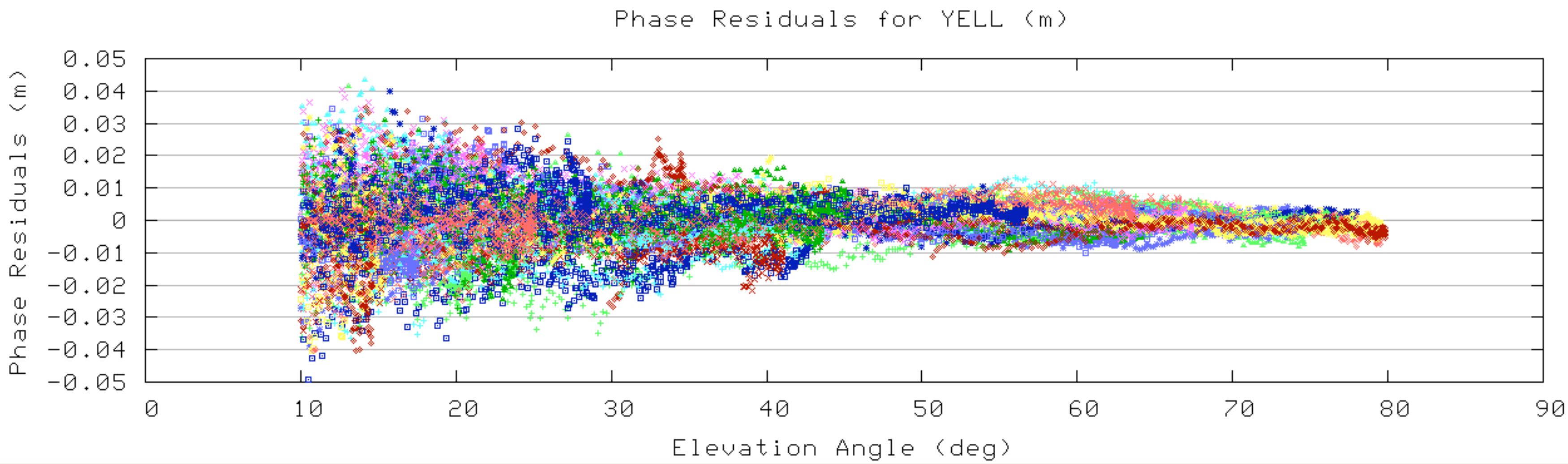
Narrowlane Ambiguities – Float & Fixed





L3 Residuals – Float & Fixed





Implications for PPP — Summary

- Each observable requires a satellite ‘clock’ parameter:
  - dt_{P3}, dt_{L3}, b_{A4}
  - as well as satellite X, Y, Z coordinates.
- In practice ($dt_{P3} - dt_{L3}$) and b_{A4} variations may allow transmission as ‘slow’ corrections.
- Standard Ambiguity Resolution Techniques (e.g. LAMBDA) become applicable to PPP.
- PPP-AR becomes possible in principle.
- ALL predicated on good orbits! (IGS Rapid here)

Conclusions

- Synchronisation of code and phase measurements is significantly different.
- The Standard Model allows the pseudorange biases to directly interfere with the carrier phase biases.
- The Decoupled Clock Model provides:
  - unambiguous, but imprecise code clock estimates
  - precise, but ambiguous phase clock estimates
  - integer ambiguities.
- Extended Model required for L1, L2 processing.
- Provides a path for PPP-AR in a very generic way
  - extension of generic LS, no a-priori bias assumptions.